Photline Technologies	**Spectral analysis of the RF phase modulation of an optical signal**
Application Note	

Description of the phase modulator:

Diffusing an optical channel waveguide at the surface of an X-cut or, more preferably, a Z-cut lithium niobate crystal obtains an optical phase modulator. The main direction of propagation can be oriented parallel to the X- or Y- axis. Cr-Au traveling wave electrodes are deposited over a thick dielectric buffer layer in order to prevent from undesirable optical absorption of the TM-mode, and also in order to get the microwave to optical phase matching condition.

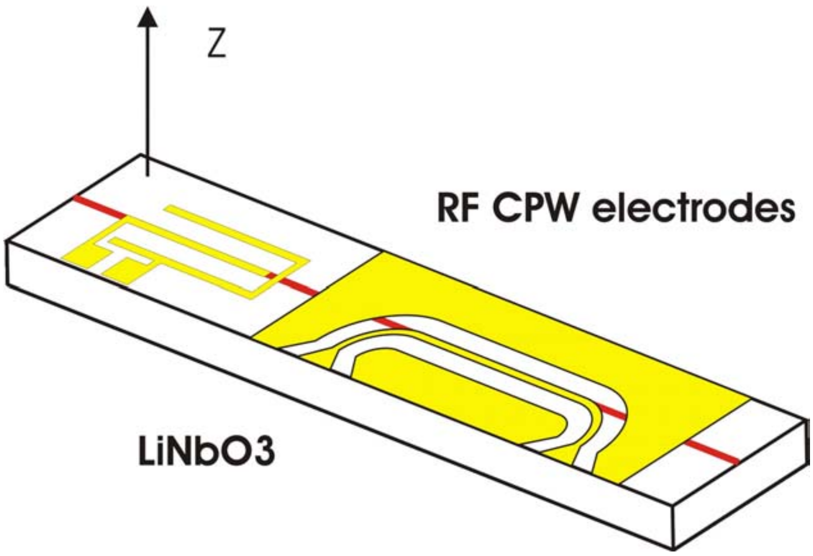

Figure 1: Scheme of a phase modulator on a Z-cut LiNbO3 crystal

Principle of the modulation:

A sinusoidal electrical signal is applied to the electrodes. The amplitude is V_z and the RF frequency pulsation is Ω.

It can be shown that the total phase variation obtained at the output of the modulator illuminated by an optical beam of wavelength λ polarized along the Z-axis, i.e. the TM polarization, is given by the following equation:

$$\phi(t) = \kappa V_Z M(\Omega) \cos(\Omega t - \psi) \qquad [1]$$

with:

$$\kappa = \frac{\pi}{\lambda G} n_e^3 r_{33} \eta L \qquad [2]$$

The frequency dependent amplitude of the signal, limited by phase mismatch and microwave losses is defined by M(Ω):

$$M(\Omega) = e^{-\left(\frac{\alpha L}{2}\right)} \left[\frac{\sinh^2\left(\frac{\alpha L}{2}\right) + \sin^2\left(\frac{\xi L}{2}\right)}{\left(\frac{\alpha L}{2}\right)^2 + \left(\frac{\xi L}{2}\right)^2} \right]^{\frac{1}{2}} \qquad [3]$$

where $\xi = \left(n_\mu - n_o\right)\frac{\Omega}{c}$, $\alpha = \alpha_o \sqrt{\Omega}$ [4]

Here, the effective microwave index is n_μ, the characteristic impedance is Z_c and the microwave loss is α. The gap of the electrode is G. the length of the electrodes is L. The electro-optic coefficient of the material is r_{33} and the optical refractive index is n_e. We also consider η as the overlap coefficient between the electric field and the optical field.

One can define at any frequency the effective half-wave voltage of a phase modulator to be:

$$V_\pi(\Omega) = \frac{\lambda G}{n_e^3 r_{33} \eta L M(\Omega)} \qquad [5]$$

At DC, the expression becomes:

$$V_\pi = \frac{\lambda G}{n_e^3 r_{33} \eta L} \qquad [6]$$

Measurement of the half-wave voltage by spectral analysis:

The measurement of the half-wave voltage at any desired working frequency can be carried by spectral analysis, through a scanning Fabry-Perot interferometer acting as a high-resolution optical spectrum analyzer (OSA). We use for this experiment a laser with a narrow line width (e.g. a 1550nm DFB laser)

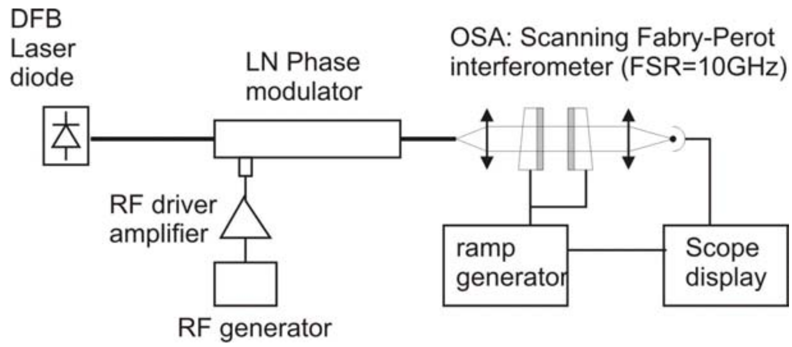

Figure 2: Experimental set-up

We first consider the general expression of the optical field, phase modulated, at the output of the modulator, for a fixed RF input frequency and power:

$$E(t) = E_o e^{j\omega t} e^{-j\gamma \cos\Omega t} \qquad [7]$$

Eo is the amplitude of the optical field. ω is the optical pulsation. γ is defined as the index of modulation:

$$\gamma = \kappa V_Z M(\Omega) \qquad [8]$$

We can now make a spectral decomposition of the signal:

$$\frac{E(t)}{E_o} = e^{j\omega t}\left[J_0(\gamma) + 2\sum_{p=1}^{\infty}(-1)^p J_{2p}(\gamma)\cos(2p\Omega t) + 2j\sum_{p'=1}^{\infty}(-1)^{p'} J_{2p'-1}(\gamma)\cos((2p'-1)\Omega t)\right]$$

In this expression, $J_n(\gamma)$ are the Bessel functions. The following figure gives the evolution of the 2 first orders of this family of function.

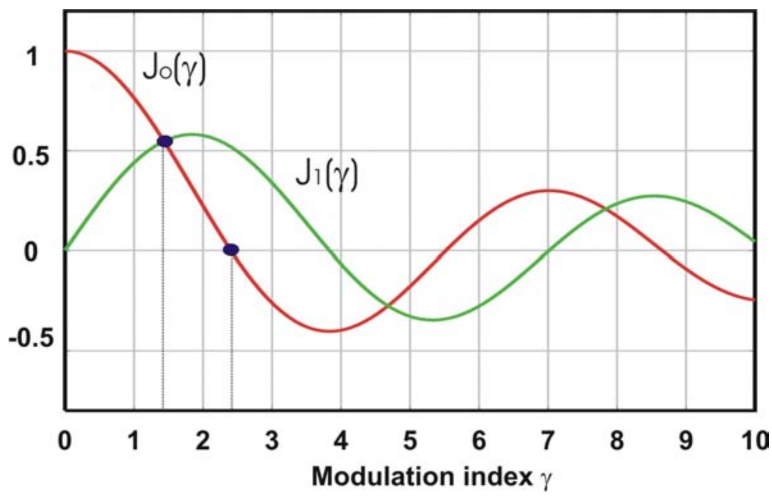

Figure 3: Jo(γ) and J1(γ)

Hence, the spectral analysis of the optical signal transmitted by a phase modulator driven at a fixed pulsation Ω and with a fixed RF input power yields a spectrum with multiple lines centered around the optical pulsation ω, each line being spaced of Ω from its neighbors. From the signal photo-detected at the output of the interferometer, the amplitude of the central line is given, in relative intensity, by $J_0^2(\gamma)$, and the two lateral ones by $J_1^2(\gamma)$.

There are two remarkable points, which correspond to the following situations:

$J_0(2.4) = 0$

and

$J_0(\gamma) = J_1(\gamma)$ for γ=1.435

These two situations are represented in figures 3 and 4.

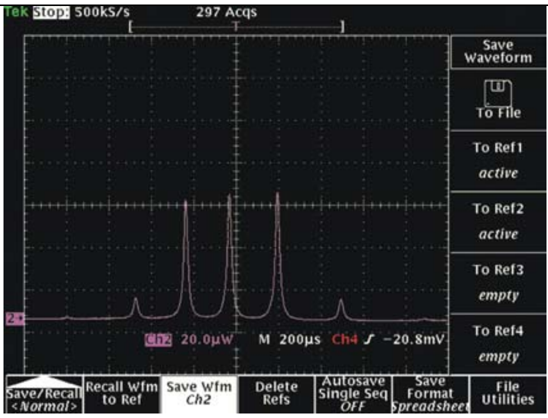	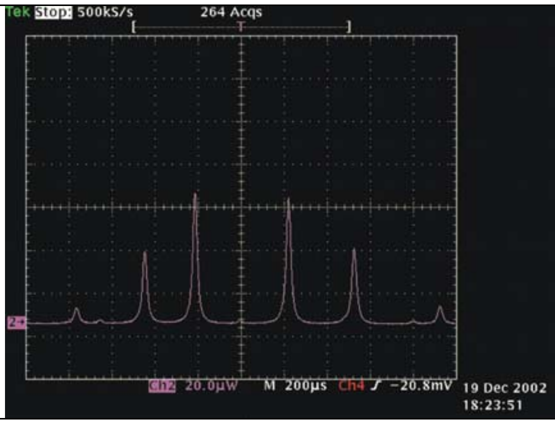
Figure 3: F=1GHz, RF input power:20.5 dBm (Jo=J1)	*Figure 4: F=1GHz, RF input power:24.6 dBm (Jo=0)*

We can use this property to calculate accurately the half-wave voltage at the measured frequency:

In the example of the figure, P=24.6dBm and f=1GHz.

From the RF input power, we get the amplitude of the driving voltage:

Vz=5.37V

We can then deduce the half wave voltage by replacing the modulation index by its value γ=2.4:

$$V_\pi(1\text{GHz}) = \frac{\pi}{2.4} V_Z = 7\text{V}$$